Technical Notes

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Dusty Hypersonic Flow past Thick Wedges

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Introduction

RESEARCH investigations on the motion of dust particles interacting with gas in an inviscid hypersonic shock layer were initiated by the contributions of Probstein and Fassio.[1] Their work was quickly extended to provide complete solutions for all flow quantities, as well as the fluid and particle trajectories. More general body shapes were also considered.[2-5] However, all of these studies assumed that the mass fraction of suspended particles is so small that the particles do not significantly affect the gas motion. One is then faced with the relatively simple problem of determining the particle motion through a known gas field.

The effects of the dust particles on the hypersonic gas flow over a slender airfoil have been considered previously by the authors.[6] In this Note, we extend the method of Ref. 6 to the case of a thick airfoil with an attached shock wave. Detailed equations are given in Ref. 7, while explicit results for the wedge are presented herein.

Governing Equations

Considering the gas and dust particle phases as two interacting continua, the equations[6,7] governing steady plane dusty gas flow are, in nondimensional variables,

$$\nabla \cdot (\rho u) = 0, \qquad \rho u \cdot \nabla u = -\nabla p + k_1 N(v - u)$$

$$\frac{\rho}{\gamma - 1} u \cdot \nabla \left(\frac{p}{\rho}\right) + p \nabla \cdot u = D_1 N\left(T_p - \gamma M_\infty^2 \frac{p}{\rho}\right) + k_1 N |v - u|^2$$

$$\nabla \cdot (Nv) = 0, \qquad v \cdot \nabla v = -k_0 (v - u)$$

$$v \cdot \nabla T_p = -D_0 \left(T_p - \gamma M_\infty^2 \frac{p}{\rho}\right)$$

where $\nabla = (\partial/\partial x,\ \partial/\partial y)$, all dependent variables have been made dimensionless by dividing by their freestream values, and independent variables x,y are nondimensionalized using a characteristic length L. Quantities k_0, k_1, D_0, D_1, are parameters related to the strength of the interaction between the two phases. The notation of Ref. 6 has been used in these equations.

When a hypersonic stream of dusty gas impinges upon a symmetric airfoil at zero incidence the boundary conditions associated with this system of equations are the Rankine-Hugoniot conditions across the shock wave (attached), the condition of no disturbance far upstream, and the requirement of flow tangency at the body surface.[6] The dust particle properties are assumed to be continuous across the shock.

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Newtonian Approximation

Before introducing the Newtonian approximation it is convenient to define new variables y^* and u_2^* by

$$y^* = y - F(x), \qquad u_2^* = u_2 - u_1 F'(x)$$

so that the tangency condition at the surface becomes $u_2^* = 0$ at $y^* = 0$.

For the Newtonian approximation we define a parameter

$$\epsilon = (\gamma - 1)/(\gamma + 1)$$

and consider the limiting process $\epsilon \to 0$, $M_\infty \to \infty$ such that $H \equiv \epsilon^{-1} M_\infty^{-2}$ remains fixed. The independent variable y^* is stretched according to $\tilde{y} = y^*/\epsilon$ and the flow variables are assumed to have the following asymptotic expansion:

$$\rho = (1/\epsilon)\rho^{(1)} + \rho^{(2)} + \ldots, \qquad u_1 = u_1^{(1)} + \epsilon u_1^{(2)} + \ldots$$

$$u_2^* = \epsilon u_2^{(1)} + \epsilon^2 u_2^{(2)} + \ldots$$

$$p = p^{(1)} + \epsilon p^{(2)} + \ldots, \qquad N = N^{(1)} + \epsilon N^{(2)} + \ldots$$

$$v_1 = v_1^{(1)} + \epsilon v_1^{(2)} + \ldots, \qquad T_p = T_p^{(1)} + \epsilon T_p^{(2)} + \ldots$$

The variables on the left-hand side of these equations are functions of x,y^* and parameters γ, M_∞, while those on the right are functions of $x,\tilde{y}$ and parameter H. These expansions are substituted into the flow equations and like powers of ϵ are equated. The corresponding boundary conditions are obtained in the same manner.[7]

Wedge Solution

For the wedge-shaped airfoil, defined by $F(x) = Ax$, the complete first-order solution is

$$u_1^{(1)} = (A^2 + 1)^{-1}, \quad u_2^{(1)} = 0, \quad p^{(1)} = A^2 (A^2 + 1)^{-1}$$

$$\rho^{(1)} = A^2 \{A^2 + H(A^2 + 1)\}^{-1}, \quad v_1^{(1)} = N^{(1)} = T_p^{(1)} = 1$$

$$v_2^{(1)} = k_0 (A^2 + 1)^{-1} [g(x) - \tilde{y}]$$

where the equation for the shock wave has been taken in the form

$$\tilde{y} = g(x) + \epsilon g_1(x) + \ldots$$

with $g(x) = A^{-1}(A^2 + 1)[H(A^2 + 1) + A^2]x$.

Using these results, the equations corresponding to the next order can be integrated to yield the complete second-order solution

$$u_1^{(2)} = -[A^2 + H(A^2 + 1)](A^2 + 1)^{-1}$$

$$u_2^{(2)} = -\{k_1 A^{-2} [H(A^2 + 1) - A^2] + 2D_1 H^{-1}\}\tilde{y}$$

$$p^{(2)} = Ak_1 (A^2 + 1)^{-1} [g(x) - \tilde{y}] + 2H + A^2 (A^2 + 1)^{-1}$$

$$\rho^{(2)} = A[H(A^2+1)+A^2]^{-2}\{k_1[H(A^2+1)-A^2] + 2D_1A^2H^{-1}\}\{g(x)-\tilde{y}\} + HA[H(A^2+1)+A^2]^{-2} \times\{2HA^3+4HA+2HA^{-1}+3A^3+3A\}$$

$$v_1^{(2)} = -Ak_0(A^2+1)^{-1}[g(x)-\tilde{y}]$$

$$v_2^{(2)} = -k_0A^{-2}(2A^2+1)(A^2+1)^{-1}[H(A^2+1)+A^2][g(x)-\tilde{y}] + k_0(A^2+1)^{-1}g_1(x)$$

$$N^{(2)} = k_0A^{-1}[g(x)-\tilde{y}]$$

$$T_p^{(2)} = D_0AH^{-1}(A^2+1)^{-1}[g(x)-\tilde{y}]$$

where

$$g_1(x) = (A^2+1)A^{-1}\{A^{-2}[A^4-H^2(A^2+1)^2] + 2[H(A^2+1)+A^2]^2 - H(A^2+1)\}x - (2A)^{-1}(A^2+1)^2[H(A^2+1)+A^2] \times\{2D_1H^{-1}+k_1A^{-2}[H(A^2+1)-A^2]\}x^2$$

Discussion

1) Expressions for the fluid streamlines and dust particle paths[7] can be obtained in the same manner as for the small-disturbance theory.[6]

2) The shock wave is bent toward the wedge surface [quadratic term in $g_1(x)$] under the influence of the dust particles.

3) Of practical interest, the surface pressure p_s on the wedge is given to second order as

$$p_s = A^2(A^2+1)^{-1} + \epsilon\{2H + A^2(A^2+1)^{-1} + k_1[H(A^2+1)+A^2]x\}$$

showing that it increases linearly with distance from the vertex of the wedge.

Acknowledgment

This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grant A4484.

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A "Similarity" Solution for Laminar Swirling Core Flows

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Introduction

THE present investigation develops a linearized "similarity" solution, based on the Oseen assumption of a small axial disturbance velocity, for the far downstream region of swirling laminar core flows embedded in a uniform parallel following stream. The solution includes the cases of "weak" swirl where the pressure perturbation is sufficiently small for the axial and azimuthal fields to be effectively uncoupled; "strong" swirl with significant pressure coupling of these fields; and the special case of "very strong" swirl in jets transmitting zero, or at least very small flow force.

The approach used to obtain the linearized solution is similar to that employed by Batchelor[1] in obtaining a similarity solution for strong trailing vortices far downstream of their point of generation.

The swirling core flows considered are of interest in connection with modeling such practical applications as the flow downstream of rotating bodies (e.g., spinning projectiles and turbomachinery). These flows will be turbulent, whether in a real application or in the laboratory; nevertheless, it appears reasonable to investigate the laminar flow, particularly from the viewpoint of understanding the role of the swirl-induced pressure coupling and its effect on the axial flow.

"Similarity" Solutions for Swirling Laminar Core Flows Far Downstream

We shall consider a single axisymmetric swirling core and use cylindrical polar coordinates (r,ϕ,z) with corresponding velocity components (u,v,w) to investigate viscous development of the core in a uniform outer stream of velocity W_0 and pressure p_∞. If we consider the steady motion of a viscous incompressible flow, and assume that $\partial/\partial z \ll \partial/\partial r$; $u \ll w$, and $|w-W_0| \ll W_0$; then the equations of motion can be approximated by[1]:

$$\frac{\partial}{\partial z}(rw) + \frac{\partial}{\partial r}(ru) = 0 \tag{1}$$

$$W_0\frac{\partial v}{\partial z} = \nu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2}\right) \tag{2}$$

$$W_0\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \tag{3}$$

$$\frac{v^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r} \tag{4}$$

where p is the disturbance pressure due to the core flow, ν the kinematic viscosity, and ρ the uniform density of the fluid.

Received June 23, 1982; revision received Nov. 23, 1982. Copyright © American Institute of Aeronautics and Astronautics, Inc., 1983. All rights reserved.

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